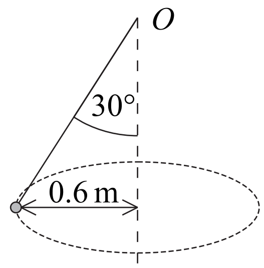
AQA Maths M2

Topic Questions from Papers

Circular Motion

1 A particle, of mass 2 kg, is attached to one end of a light inextensible string. The other end is fixed to the point O. The particle is set into motion, so that it describes a horizontal circle of radius 0.6 metres, with the string at an angle of 30° to the vertical. The centre of the circle is vertically below O.

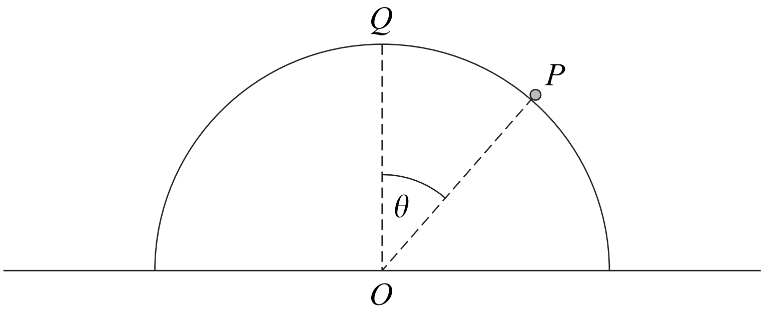


(a) Show that the tension in the string is 22.6 N, correct to three significant figures. *(3 marks)*

(b) Find the speed of the particle. *(4 marks)*

(Q2, Jan 2006)

2 A particle P, of mass m kg, is placed at the point Q on the top of a smooth upturned hemisphere of radius 3 metres and centre O. The plane face of the hemisphere is fixed to a horizontal table. The particle is set into motion with an initial horizontal velocity of $2\,\mathrm{m\,s^{-1}}$. When the particle is on the surface of the hemisphere, the angle between OP and OQ is θ and the particle has speed $v\,\mathrm{m\,s^{-1}}$.

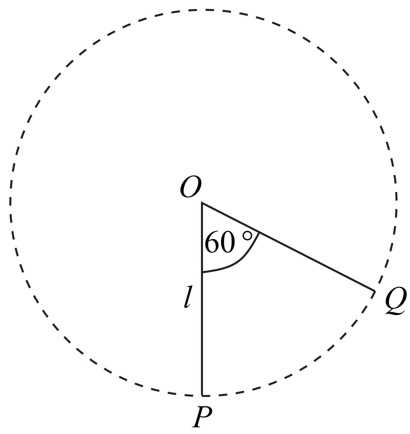


(a) Show that $v^2 = 4 + 6g(1 - \cos\theta)$. *(4 marks)*

(b) Find the value of θ when the particle leaves the hemisphere. *(5 marks)*

(Q6, Jan 2006)

3 A particle of mass m is suspended from a fixed point O by a light inextensible string of length l. The particle hangs in equilibrium at the point P vertically below O. The particle is then set into motion with a horizontal velocity U so that it moves in a complete vertical circle with centre O. The point Q on the circle is such that $\angle POQ = 60°$, as shown in the diagram.



(a) Find, in terms of g, l and U, the speed of the particle at Q. *(4 marks)*

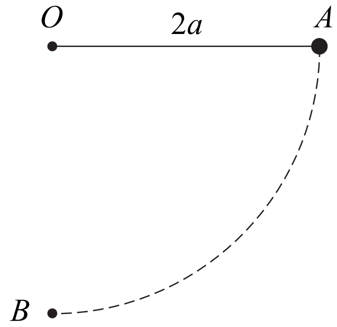
(b) Find, in terms of g, l, m and U, the tension in the string when the particle is at Q.
(5 marks)
(Q4, June 2006)

4 A car of mass 1200 kg travels round a roundabout on a horizontal, circular path at a constant speed of $14\,\text{m s}^{-1}$. The radius of the circle is 50 metres. Assume that there is no resistance to the motion of the car and that the car can be modelled as a particle.

(a) A friction force, directed towards the centre of the roundabout, acts on the car as it moves. Show that the magnitude of this friction force is 4704 N. *(4 marks)*

(b) The coefficient of friction between the car and the road is μ. Show that $\mu \geqslant 0.4$.
(3 marks)
(Q5, June 2006)

5 A light inextensible string has length $2a$. One end of the string is attached to a fixed point O and a particle of mass m is attached to the other end. Initially, the particle is held at the point A with the string taut and horizontal. The particle is then released from rest and moves in a circular path. Subsequently, it passes through the point B, which is directly below O. The points O, A and B are as shown in the diagram.

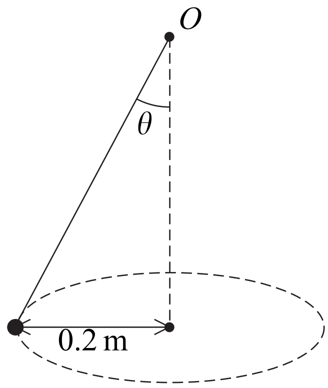


(a) Show that the speed of the particle at B is $2\sqrt{ag}$. *(3 marks)*

(b) Find the tension in the string as the particle passes through B. Give your answer in terms of m and g. *(3 marks)*

(Q3, Jan 2007)

6 A particle is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle is set into motion, so that it describes a horizontal circle whose centre is vertically below O. The angle between the string and the vertical is θ, as shown in the diagram.



(a) The particle completes 40 revolutions every minute.

Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

(b) The radius of the circle is 0.2 metres.

Find, in terms of π, the magnitude of the acceleration of the particle. *(2 marks)*

(c) The mass of the particle is m kg and the tension in the string is T newtons.

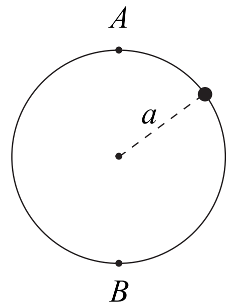
(i) Draw a diagram showing the forces acting on the particle. *(1 mark)*

(ii) Explain why $T\cos\theta = mg$. *(1 mark)*

(iii) Find the value of θ, giving your answer to the nearest degree. *(5 marks)*

(Q6, Jan 2007)

7 A bead of mass m moves on a smooth circular ring of radius a which is fixed in a vertical plane, as shown in the diagram. Its speed at A, the highest point of its path, is v and its speed at B, the lowest point of its path, is $7v$.



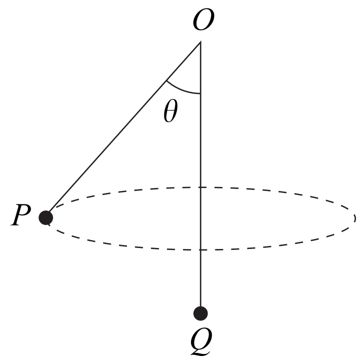
(a) Show that $v = \sqrt{\dfrac{ag}{12}}$. *(5 marks)*

(b) Find the reaction of the ring on the bead, in terms of m and g, when the bead is at A. *(4 marks)*

(Q5, June 2007)

8 A particle, P, of mass 3 kg is attached to one end of a light inextensible string. The string passes through a smooth fixed ring, O, and a second particle, Q, of mass 5 kg is attached to the other end of the string. The particle Q hangs at rest vertically below the ring and the particle P moves with speed $4\,\text{m}\,\text{s}^{-1}$ in a horizontal circle, as shown in the diagram.

The angle between OP and the vertical is θ.



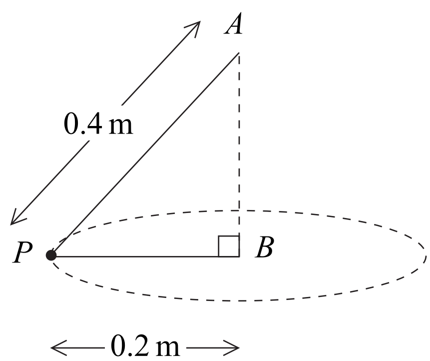
(a) Explain why the tension in the string is 49 N. *(2 marks)*

(b) Find θ. *(3 marks)*

(c) Find the radius of the horizontal circle. *(4 marks)*

(Q8, June 2007)

9 Two light inextensible strings, of lengths 0.4 m and 0.2 m, each have one end attached to a particle, P, of mass 4 kg. The other ends of the strings are attached to the points A and B respectively. The point A is vertically above the point B. The particle moves in a horizontal circle, centre B and radius 0.2 m, at a speed of $2\,\text{m}\,\text{s}^{-1}$. The particle and strings are shown in the diagram.



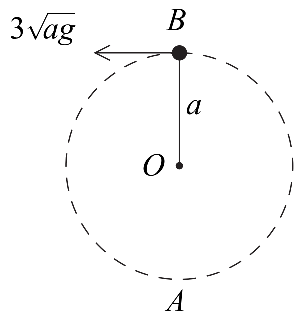
(a) Calculate the magnitude of the acceleration of the particle. *(2 marks)*

(b) Show that the tension in string PA is 45.3 N, correct to three significant figures. *(4 marks)*

(c) Find the tension in string PB. *(3 marks)*

(Q5, Jan 2008)

10 A light inextensible string, of length a, has one end attached to a fixed point O. A particle, of mass m, is attached to the other end. The particle is moving in a vertical circle, centre O. When the particle is at B, vertically above O, the string is taut and the particle is moving with speed $3\sqrt{ag}$.

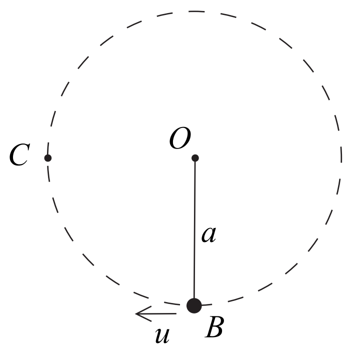


(a) Find, in terms of g and a, the speed of the particle at the lowest point, A, of its path. *(4 marks)*

(b) Find, in terms of g and m, the tension in the string when the particle is at A. *(4 marks)*

(Q7, Jan 2008)

11 A small bead, of mass m, is suspended from a fixed point O by a light inextensible string, of length a. The bead is then set into circular motion with the string taut at B, where B is vertically below O, with a horizontal speed u.



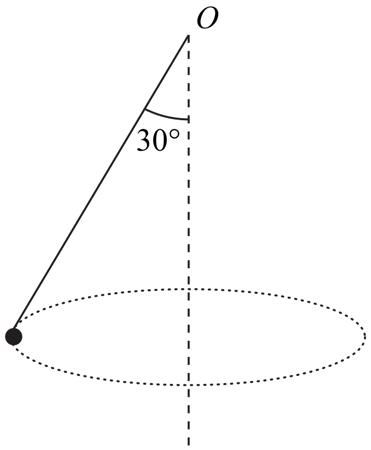
(a) Given that the string does not become slack, show that the least value of u required for the bead to make complete revolutions about O is $\sqrt{5ag}$. *(5 marks)*

(b) In the case where $u = \sqrt{5ag}$, find, in terms of g and m, the tension in the string when the bead is at the point C, which is at the same horizontal level as O, as shown in the diagram. *(3 marks)*

(c) State one modelling assumption that you have made in your solution. *(1 mark)*

(Q7, June 2008)

12 A particle, of mass 6 kg, is attached to one end of a light inextensible string. The other end of the string is attached to the fixed point O. The particle is set in motion, so that it moves in a horizontal circle at constant speed, with the string at an angle of 30° to the vertical. The centre of this circle is vertically below O.



The particle moves in a horizontal circle with an angular speed of 40 revolutions per minute.

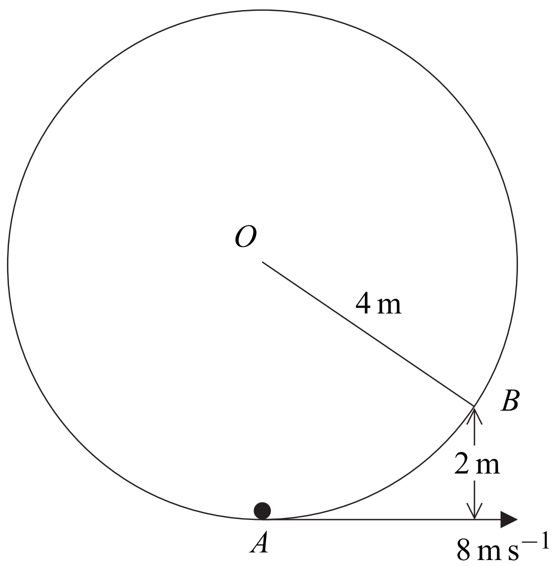
(a) Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

(b) Show that the tension in the string is 67.9 N, correct to three significant figures. *(3 marks)*

(c) Find the radius of the horizontal circle. *(4 marks)*

(Q5, Jan 2009)

13 A hollow cylinder, of internal radius 4 m, is fixed so that its axis is horizontal. The point O is on this axis. A particle, of mass 6 kg, is set in motion so that it moves on the smooth **inner** surface of the cylinder in a vertical circle about O. Its speed at the point A, which is vertically below O, is $8\,\mathrm{m\,s^{-1}}$.



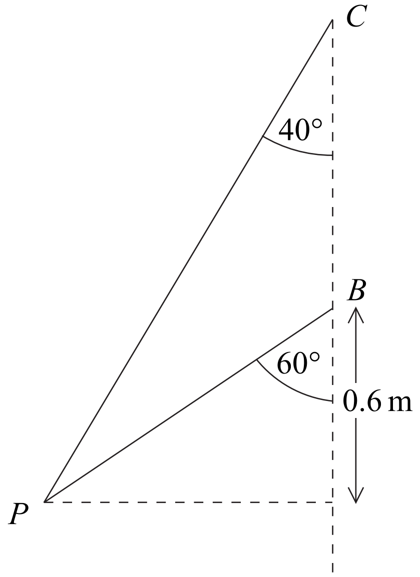
When the particle is at the point B, at a height of 2 m above A, find:

(a) its speed; *(3 marks)*

(b) the normal reaction between the cylinder and the particle. *(4 marks)*

(Q7, Jan 2009)

14 Two light inextensible strings each have one end attached to a particle, P, of mass 6 kg. The other ends of the strings are attached to the fixed points B and C. The point C is vertically above the point B. The particle moves, at constant speed, in a horizontal circle, with centre 0.6 m below point B, with the strings inclined at 40° and 60° to the vertical, as shown in the diagram. Both strings are taut.



(a) As the particle moves in the horizontal circle, the tensions in the two strings are equal.

Show that the tension in the strings is 46.4 N, correct to three significant figures. *(4 marks)*

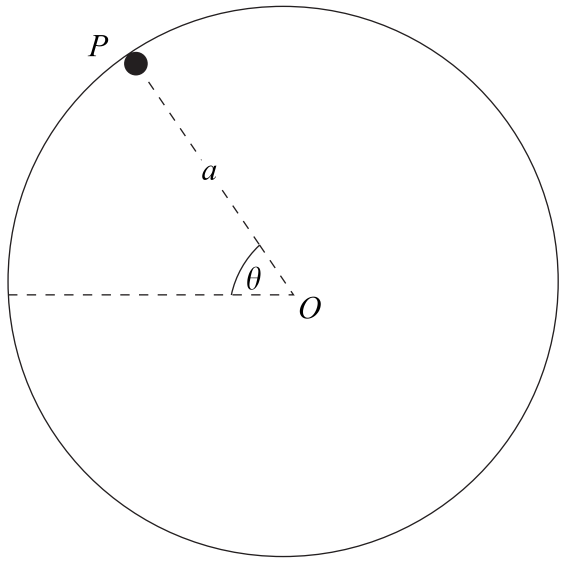
(b) Find the speed of the particle. *(4 marks)*

(Q4, June 2009)

15 In crazy golf, a golf ball is hit so that it starts to move in a vertical circle on the inside of a smooth cylinder.

Model the golf ball as a particle, P, of mass m. The circular path of the golf ball has radius a and centre O. At time t, the angle between OP and the horizontal is θ, as shown in the diagram.

The golf ball has speed u at the lowest point of its circular path.



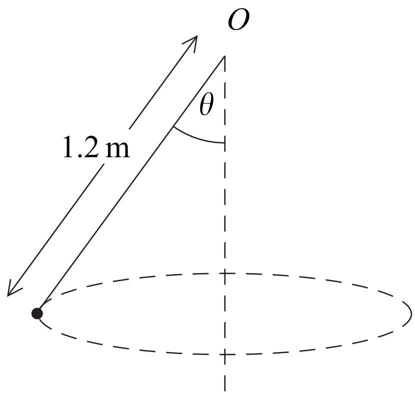
(a) Show that, while the golf ball is in contact with the cylinder, the reaction of the cylinder on the golf ball is

$$\frac{mu^2}{a} - 3mg\sin\theta - 2mg$$ *(6 marks)*

(b) Given that $u = \sqrt{3ag}$, the golf ball will not complete a vertical circle inside the cylinder. Find the angle which OP makes with the horizontal when the golf ball leaves the surface of the cylinder. *(4 marks)*

(Q7, June 2009)

16 A particle, of mass 4 kg, is attached to one end of a light inextensible string of length 1.2 metres. The other end of the string is attached to a fixed point O. The particle moves in a horizontal circle at a constant speed. The angle between the string and the vertical is θ.



(a) Find the radius of the horizontal circle in terms of θ. *(1 mark)*

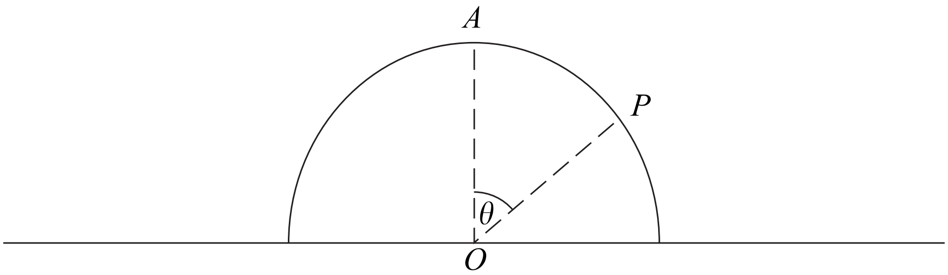
(b) The angular speed of the particle is 5 radians per second. Find θ. *(6 marks)*

(Q6, Jan 2010)

17 A smooth hemisphere, of radius a and centre O, is fixed with its plane face on a horizontal surface. A particle, of mass m, can move freely on the surface of the hemisphere.

The particle is placed at the point A, the highest point of the hemisphere, and is set in motion along the surface with speed u.

(a) While the particle is in contact with the hemisphere at a point P, OP makes an angle θ with the upward vertical.



Show that the speed of the particle at P is

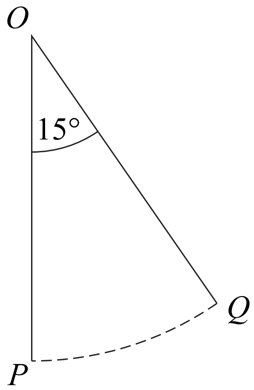
$$\left(u^2 + 2ga[1 - \cos\theta]\right)^{\frac{1}{2}}$$ *(5 marks)*

(b) The particle leaves the surface of the hemisphere when $\theta = \alpha$.

Find $\cos\alpha$ in terms of a, u and g. *(5 marks)*

(Q7, Jan 2010)

18 A particle is attached to one end of a light inextensible string of length 3 metres. The other end of the string is attached to a fixed point O. The particle is set into motion horizontally at point P with speed v, so that it describes part of a vertical circle whose centre is O. The point P is vertically below O.



The particle first comes momentarily to rest at the point Q, where OQ makes an angle of 15° to the vertical.

(a) Find the value of v. *(4 marks)*

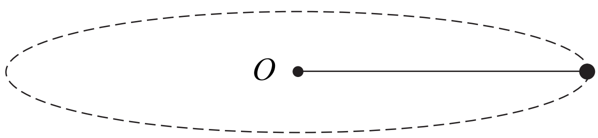
(b) When the particle is at rest at the point Q, the tension in the string is 22 newtons.

Find the mass of the particle. *(3 marks)*

(Q8, June 2010)

19 A particle, of mass 8 kg, is attached to one end of a length of elastic string. The particle is placed on a smooth horizontal surface. The other end of the elastic string is attached to a point O fixed on the horizontal surface.

The elastic string has natural length 1.2 m and modulus of elasticity 192 N.



The particle is set in motion on the horizontal surface so that it moves in a circle, centre O, with constant speed $3\,\text{m}\,\text{s}^{-1}$.

Find the radius of the circle. *(8 marks)*

(Q9, June 2010)

20 (a) A shiny coin is on a rough horizontal turntable at a distance 0.8 m from its centre. The turntable rotates at a constant angular speed. The coefficient of friction between the shiny coin and the turntable is 0.3 .

Find the maximum angular speed, in radians per second, at which the turntable can rotate if the shiny coin is not going to slide. *(4 marks)*

(b) The turntable is stopped and the shiny coin is removed. An old coin is placed on the turntable at a distance 0.15 m from its centre. The turntable is made to rotate at a constant angular speed of 45 revolutions per minute.

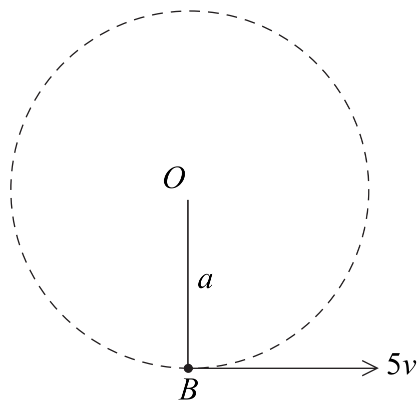
(i) Find the angular speed of the turntable in radians per second. *(2 marks)*

(ii) The old coin remains in the same position on the turntable.

Find the least value of the coefficient of friction between the old coin and the turntable needed to prevent the old coin from sliding. *(4 marks)*

(Q5, Jan 2011)

21 A light inextensible string, of length a, has one end attached to a fixed point O. A small bead, of mass m, is attached to the other end of the string. The bead is moving in a vertical circle, centre O. When the bead is at B, vertically below O, the string is taut and the bead is moving with speed $5v$.



(a) The speed of the bead at the highest point of its path is $3v$.

Find v in terms of a and g. *(4 marks)*

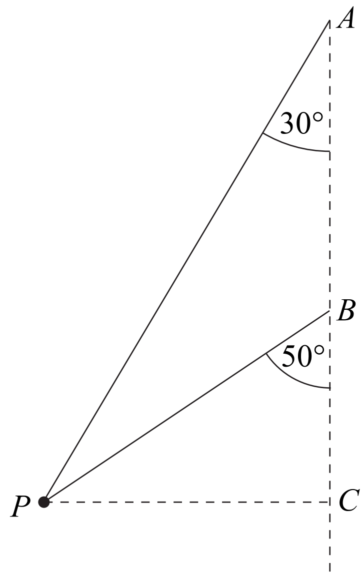
(b) Find the ratio of the greatest tension to the least tension in the string, as the bead travels around its circular path. *(5 marks)*

(Q6, Jan 2011)

22 Two light inextensible strings each have one end attached to a particle, P, of mass 4 kg. The other ends of the strings are attached to the fixed points A and B. The point A is vertically above the point B.

The particle moves at a constant speed in a horizontal circle. The centre, C, of this circle is directly below the point B. The two strings are inclined at 30° and 50° to the vertical, as shown in the diagram. Both strings are taut.

As the particle moves in the horizontal circle, the tension in the string BP is 20 N.



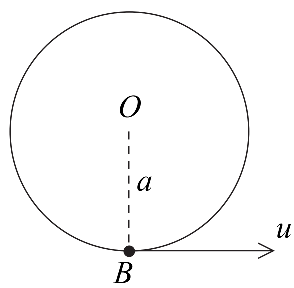
(a) Find the tension in the string AP. *(4 marks)*

(b) The speed of the particle is $5\,\text{m s}^{-1}$.

Find the length of CP, the radius of the horizontal circle. *(4 marks)*

(Q7, June 2011)

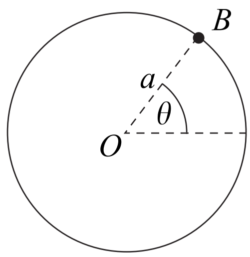
23 A smooth wire is fixed in a vertical plane so that it forms a circle of radius a metres and centre O. A bead, B, of mass 0.3 kg, is threaded on the wire and is set in motion with a speed $u\,\text{m s}^{-1}$ at the lowest point of its circular path, as shown in the diagram.



(a) Show that, if the bead is going to make complete revolutions around the wire,

$$u > 2\sqrt{ag}$$ *(3 marks)*

(b) At time t seconds, the angle between OB and the horizontal is θ, as shown in the diagram.



It is given that $u = \sqrt{\frac{9}{2}ag}$.

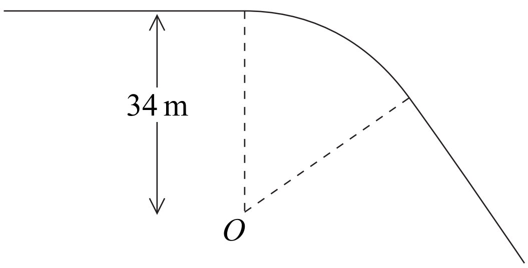
(i) Find the reaction of the bead on the wire, giving your answer in terms of g and θ. *(5 marks)*

(ii) Find θ when this reaction is zero. *(2 marks)*

(Q8, June 2011)

24 A parcel is placed on a flat rough horizontal surface in a van. The van is travelling along a horizontal road. It travels around a bend of radius 34 m at a constant speed. The coefficient of friction between the parcel and the horizontal surface in the van is 0.85.

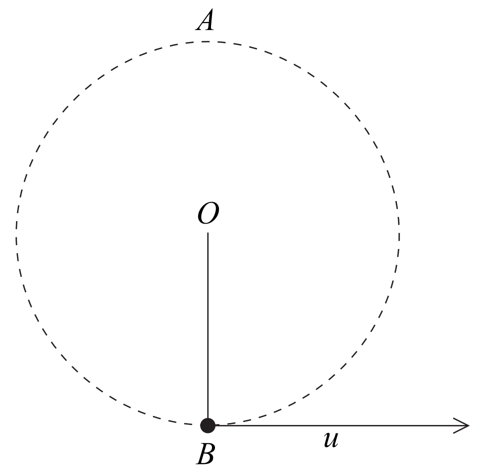
Model the parcel as a particle travelling around part of a circle of radius 34 m and centre O, as shown in the diagram.



Find the greatest speed at which the van can travel around the bend without causing the parcel to slide. *(6 marks)*

(Q5, Jan 2012)

25 A small bead, of mass m, is suspended from a fixed point O by a light inextensible string of length a. With the string taut, the bead is at the point B, vertically below O, when it is set into vertical circular motion with an initial horizontal velocity u, as shown in the diagram.



The string does not become slack in the subsequent motion. The velocity of the bead at the point A, where A is vertically above O, is v.

(a) Show that $v^2 = u^2 - 4ag$. *(2 marks)*

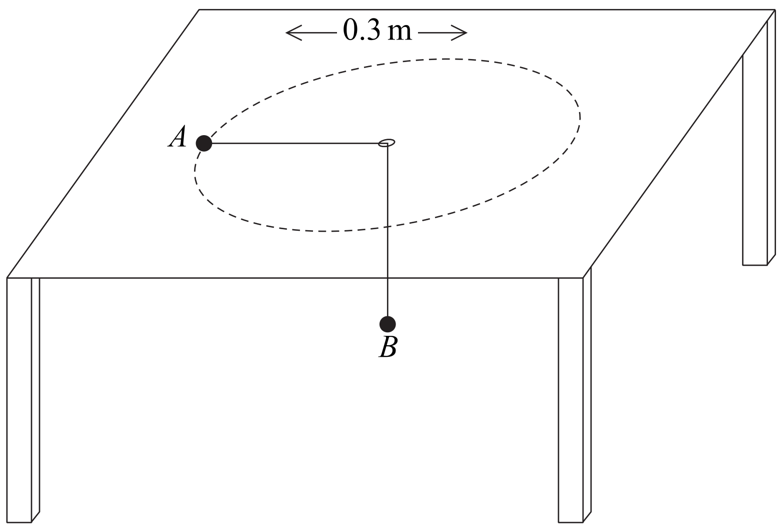
(b) The ratio of the tensions in the string when the bead is at the two points A and B is $2:5$.

(i) Find u in terms of g and a. *(7 marks)*

(ii) Find the ratio $u:v$. *(2 marks)*

(Q7, Jan 2012)

26 Two particles, A and B, are connected by a light inextensible string which passes through a hole in a smooth horizontal table. The edges of the hole are also smooth. Particle A, of mass 1.4 kg, moves, on the table, with constant speed in a circle of radius 0.3 m around the hole. Particle B, of mass 2.1 kg, hangs in equilibrium under the table, as shown in the diagram.



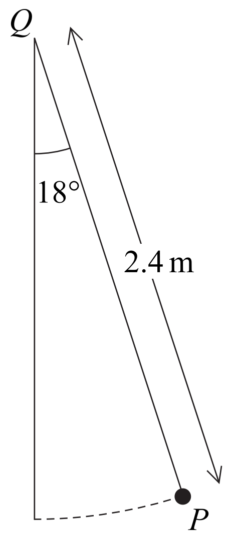
(a) Find the angular speed of particle A. *(4 marks)*

(b) Find the speed of particle A. *(2 marks)*

(c) Find the time taken for particle A to complete one full circle around the hole. *(2 marks)*

(Q5, June 2012)

27 Simon, a small child of mass 22 kg, is on a swing. He is swinging freely through an angle of 18° on both sides of the vertical. Model Simon as a particle, P, of mass 22 kg, attached to a fixed point, Q, by a light inextensible rope of length 2.4 m.



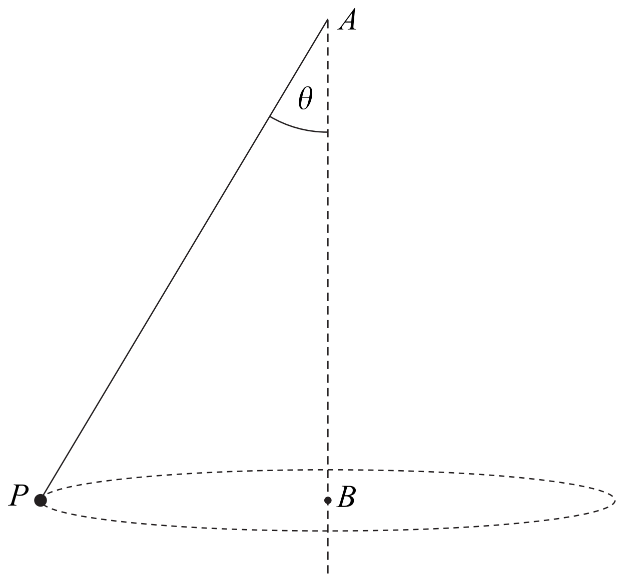
(a) Find Simon's maximum speed as he swings. *(4 marks)*

(b) Calculate the tension in the rope when Simon's speed is a maximum. *(3 marks)*

(Q6, June 2012)

28 A light inextensible string has one end attached to a particle, P, of mass 2 kg. The other end of the string is attached to the fixed point A. The point A is vertically above the point B. The particle moves at a constant speed in a horizontal circle of radius 0.8 m and centre B. The tension in the string is 34 N.

The string is inclined at an angle θ to the vertical, as shown in the diagram.



(a) Find the angle θ. *(3 marks)*

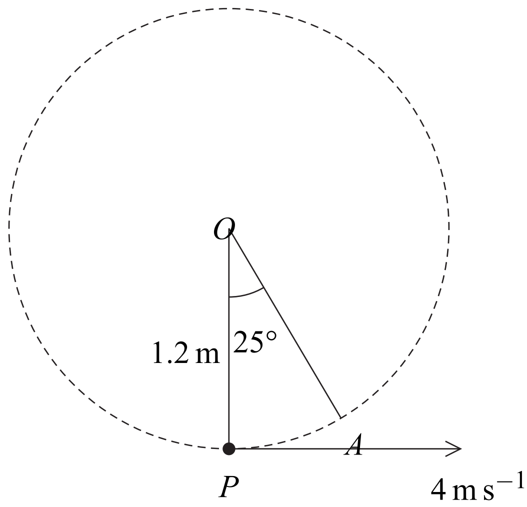
(b) Find the speed of the particle. *(3 marks)*

(c) Find the time taken for the particle to make one complete revolution. *(2 marks)*

(Q6, Jan 2013)

29 A small ball, of mass 3 kg, is suspended from a fixed point O by a light inextensible string of length 1.2 m. Initially, the string is taut and the ball is at the point P, vertically below O. The ball is then set into motion with an initial horizontal velocity of $4\,\mathrm{m\,s^{-1}}$.

The ball moves in a vertical circle, centre O. The point A, on the circle, is such that angle AOP is 25°, as shown in the diagram.



(a) Find the speed of the ball at the point A. *(4 marks)*

(b) Find the tension in the string when the ball is at the point A. *(3 marks)*

(Q7, Jan 2013)

30 Tom is travelling on a train which is moving at a constant speed of $15\,\mathrm{m\,s^{-1}}$ on a horizontal track. Tom has placed his mobile phone on a rough horizontal table. The coefficient of friction between the phone and the table is 0.2.

The train moves round a bend of constant radius. The phone does not slide as the train travels round the bend.

Model the phone as a particle moving round part of a circle, with centre O and radius r metres.

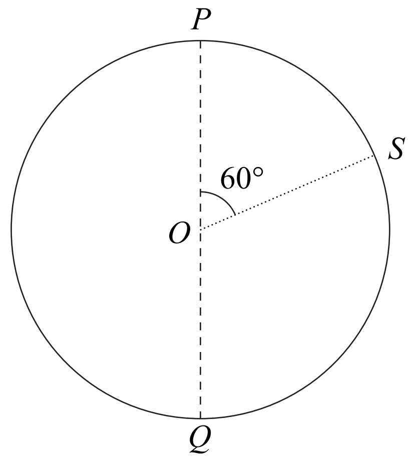
Find the least possible value of r. *(4 marks)*

(Q5, June 2013)

31 A bead, of mass m, moves on a smooth circular ring, of radius a and centre O, which is fixed in a vertical plane. At P, the highest point on the ring, the speed of the bead is $2u$; at Q, the lowest point on the ring, the speed of the bead is $5u$.

(a) Show that $u = \sqrt{\dfrac{4ag}{21}}$. *(4 marks)*

(b) S is a point on the ring so that angle POS is $60°$, as shown in the diagram.



Find, in terms of m and g, the magnitude of the reaction of the ring on the bead when the bead is at S. *(5 marks)*

(Q8, June 2013)